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ABSTRACT

This handbook is intended to provide mathematics curriculum leaders and supervisors with discussion and related activities central to a presentation of algebra for teachers of students with a history of lower academic achievement and who are part of our underserved populations. The handbook: (1) deals with general algebra activities; (2) focuses on the importance of placing algebra in an application context; (3) addresses necessary prior skills for students; (4) discusses expectations of and for students; (5) contains activities for the maintenance and extension of skills; (6) discusses what it takes to be successful in algebra; and (7) emphasizes that any workshop based on these materials is to be participatory. After discussing general considerations in developing a workshop and the nature of a good activity, the handbook presents ideas that can be employed to teach general ideas such as justifying the steps in a process, reversal activities, and where to start the problem. Activities to teach the concepts of sum, product, equations, and formulas are also included. Other sections include activities for review, a summary of the document, and a bibliography and resource list. (Contains 17 references.) (MDH)

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IN-SERVICE HANDBOOK

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

ALGEBRA FOR EVERYONE

IN-SERVICE HANDBOOK

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INTRODUCTION

The Mathematics Education Trust (Met) recently has emphasized the need for all students to learn the fundamental concepts of algebra regardless of academic level. This reflects the beliefs set forth in the **NCTM Curriculum and Evaluation Standards for School Mathematics,** emphasizing the need for all students to study the same basic core, varied only by depth of treatment. Responding to this need, the Mathematics Education Trust has sponsored the preparation of the book *Algebra for Everyone*, a videotape of exemplary teaching practices from the classrooms of algebra teachers, and this in-service resource handbook.

This handbook is intended for use by mathematics curriculum leaders and supervisors, and consequently reflects several assumptions about the workshop leaders who will use the material:

- Mathematics curriculum leaders know their subject well and understand important aspects of an algebra course. They also know the needs of algebra students, especially the weaker students who need to gain an understanding of the fundamental concepts of algebra if those students are to be productive in the world of work.

- Mathematics curriculum leaders will be able to take the ideas presented in this handbook and tailor an in-service program to the background and needs of teacher participants so that the philosophical aspects will be integrated with the suggested activities.

- Since mathematics curriculum leaders possess the qualities suggested above, these in-service ideas are not presented as a step-by-step chronological recipe. Good in-service leaders always develop programs according to their own style and personality.

- It is assumed that the in-service presentation will be participatory—that is, teachers will be involved in discussions of philosophical ideas and will engage in performing the activities presented in the in-service program.

WORKSHOP GOALS

The goal of this handbook is to provide curriculum leaders with discussion and related activities central to a presentation for participants who will teach students with a history of lower academic achievement and with students who are part of our underserved populations. This handbook—

- deals with general algebra activities;
- focuses on the importance of placing algebra in an application context;
- addresses necessary prior skills for students, recognizing that deficiencies in arithmetic cannot be allowed to stand in the way of algebraic learning;
- discusses expectations of and for students;
- contains activities for the maintenance and extension of skills;
- discusses what it takes to be successful in algebra; and
- emphasizes that any workshop based on these materials is to be participatory, that it involves questions and answers, and that participants are to have fun as they involve themselves in the workshop.

This in-service guide will not answer all the questions, philosophical and practical, that might arise in a workshop for algebra teachers. However, it does address important issues regarding a subject that is increasing in importance for students as our society and career possibilities change significantly.

GENERAL CONSIDERATIONS

There are some general considerations one should keep in mind as a workshop is developed and presented. Here are some points of emphasis for the leader to express and for teachers to remember as the workshop progresses.

1. Use a spiral approach in concept development and skills acquisition.

2. *Preview* algebraic ideas as well as review them.

3. Students should verbalize the nature of the task at hand in connection with the type of answer expected. (Example: The difference between solving an equation and simplifying an expression.)

4. Students should communicate (both orally and in writing) the connection between concepts and the vocabulary used.

5. The notion of "reasonableness of results" should be emphasized, whether it refers to the magnitude of a result or to the possibility of that result (such as extraneous results).

6. Cooperative learning groups should be encouraged as a teaching/learning strategy.

7. *Effective* activities can be used at any time or for the sake of variety, because they:
   - promote involvement
   - focus on concepts
   - have variety and are versatile
   - have preview/review value
   - are fun to do

8. The purpose of a worksheet or activity is to:
   - solidify concepts and processes
   - provide sequenced samples and exercises
   - contribute to the spiral approach
   - get students to summarize, apply, and demonstrate learned concepts
   - allow the teacher to present exercises for closure

9. Concept development can and should occur in a variety of ways by:
   - expressing ideas
   - comparing and contrasting
   - using vocabulary, stating relationships, and completing analogies
   - displaying and extending patterns
   - communicating, both in written and in oral form

10. Teachers need to encourage students to read aloud algebraic notation, expressions, and relationships. For example:

    x^2y^3z, $x^2 + y^3 + z$, $\sqrt{x} + \sqrt{y}$,

    $\sqrt{x} + y$, $3(x - 5) = 7$, $3x - 7 < 4x$, $2^x = 64$,

    $x^a x^b = x^{a+b}$, $\sqrt{98} = 7\sqrt{2}$, $[a + b] = [a] + [b]$

11. There are mathematical themes and concepts that the student should possess prior to beginning an algebra course. The following concepts and connections among them are prerequisite to the course:
    - concept of variable
    - number concepts
    - order of operations
    - evaluation of expressions and formulas
    - solving equations (informal and formal methods)
    - concept of squares and square roots
    - writing statements to represent given problems or patterns
    - plotting of points on a coordinate system

The activity "What and Why" supports many of these considerations.

WHAT AND WHY?

Intent: To get students to focus on concepts that support specific procedures in various algebraic settings. The activity is a response to students who ask "How did you get that second step?"

Approach: Students are to identify the key procedure used to go from step (a) to step (b) in various algorithms and to discuss why the procedure was followed.

Examples: Step a: $3(x + 5) - 4x = 2(x - 4)$ **What: I multiplied using the distributive property.**

Step b: $3x + 15 - 4x = 2x - 8$ **Why: To clear the parentheses so terms can be combined.**

Comment:

- It is not recommended that actual problems be solved. The focus is for students to be able to communicate the approach and their thinking about why their approach is useful. Students are encouraged to look at the "big picture" and not only to mimic what the teacher does at the chalkboard.

- It must be noted that this activity contains some examples that are inappropriate for beginning algebra. However, it is designed for use with workshop participants and is at a higher level. It does represent many of the qualities listed as considerations. This activity is versatile in that exercises appropriate to developmental levels of students can be found easily and can be used at various times during the course.

- This activity can be spiraled throughout the course. As new procedures (algorithms) are considered, students will have additional opportunities to communicate concepts associated with procedures. This type of activity might be beneficial in preparation for tests and final examinations.

WHAT AND WHY?

For each of the following, indicate: First . . . What was done in going from step a to step b.
Second . . . Why was it done?

1. Solve for x:

   a. $0.5x + 0.01x = 10$ **What:** **I multiplied both sides of the equation by 100.**

   b. $50x + x = 1000$ **Why:** **To clear decimals (fractions) so that I can work with whole numbers.**

2. Solve for x and y:

   a. $\begin{cases} x + 2y = 5 \\ 3x - 2y = 4 \end{cases}$ **What:** _____

   b. $4x = 9$ **Why:** _____

3. Solve for m:

   a. $m^2 + 5m + 6 = 0$ **What:** _____

   b. $(m + 3)(m + 2) = 0$ **Why:** _____

4. Solve for x:

   a. $\dfrac{x}{x+1} + \dfrac{3}{x} = 6$ **What:** _____

   b. $x(x+1)\left(\dfrac{x}{x+1}\right) + x(x+1)\left(\dfrac{3}{x}\right) = 6(x)(x+1)$ **Why:** _____

   $x^2 + 3(x+1) = 6(x)(x+1)$

5. Simplify:

   a. $\sqrt{98} + \sqrt{50}$ **What:** _____

   b. $\sqrt{49 \cdot 2} + \sqrt{25 \cdot 2}$ **Why:** _____

WHAT AND WHY?

6. Solve for x and y:

a. $\begin{cases} y = x + 2 \\ 3x + 4y = 10 \end{cases}$

What: _____

_____

b. $3x + 4y = 10$
$3x + 4(x + 2) = 10$

Why: _____

_____

7. Simplify:

a. $\dfrac{\dfrac{3}{x} + \dfrac{4}{y}}{\dfrac{2}{xy}}$

What: _____

_____

Why: _____

_____

b. $\dfrac{xy\left(\dfrac{3}{x}\right) + xy\left(\dfrac{4}{y}\right)}{xy\left(\dfrac{2}{xy}\right)} =$

$\dfrac{3y + 4x}{2}$

8. Solve for r:

a. $\dfrac{3}{r-1} = \dfrac{7}{r}$

What: _____

_____

b. $3r = 7r - 7$

Why: _____

_____

THE NATURE OF A GOOD ACTIVITY

A good activity has several elements (as previously suggested in the section on *General Considerations).* These elements are:

- student involvement
- focus on fundamental concepts
- variety and versatility
- review or preview of concepts
- enjoyability

The following activity, "Which Does Not Belong?" could be used as an example of a good activity. Once good activities are discussed and this particular activity is concluded, participants should discuss whether it meets the criteria for a good activity. Moreover, teachers always need to reflect on the content of an activity and the method of delivery. The content usually doesn't determine the effectiveness of the activity with respect to lesson objectives. However, the method of delivery has great impact on the success of the activity and the lesson.

WHICH DOES NOT BELONG?

Intent: The student should recognize similar and contrasting properties or attributes associated with sets of numbers, expressions, equations, or diagrams.

Approach: Given a set of four items, the student is asked to select the one that does not appear to belong with the other three. Students also should be expected to explain the rationale for their selection. Because of the open-ended nature of the directions, different students may present an appropriate argument for any of the four being the "oddball."

Example: 1) (a) $6x = 12$
(b) $6x = 10$
(c) $6x = 36$
(d) $6x = 60$

A typical student response would be that equation b does not belong because the other three equations have whole number solutions and $6x = 10$ does not.

2) (a) $\sqrt{98}$
(b) $\sqrt{44}$
(c) $\sqrt{17}$
(d) $\sqrt{27}$

3) (a) $x - 16$
(b) $x - 12$
(c) $x - 64$
(d) $x - 100$

4) (a) 81

(b) 8

(c) 1

(d) 1000

5) (a) $3(x + 4) = 3x + 7$

(b) $2(x + 5) = 2x + 10$

(c) $5(x + 1) = 5x + 5$

(d) $7(x + 4) = 7x + 28$

Comments: This type of activity has two features. Not only must the student explain why a particular example is the "oddball," but in doing so must explain the property the remaining items have in common to make them belong. Students (and workshop participants), working in groups, typically generate other problems of this type from many different areas of mathematics. Students will feel better about their participation and will be more motivated as well as have a chance to stump the teacher. One may set the criteria that problems be half mathematical and half outside mathematics, and allow one stumper ("stinker") in the set. As an example, "Which number does not belong? 45, 69, 75, 96?"

For Michigan residents, this would be most appropriate since all but 45 are interstate highways that pass through Michigan. Other choices could be chosen for other localities.

A super challenge is to create an example where each of the items can be shown to be the "oddball" under different rationales. (This activity is not restricted to algebra. It may be used in any mathematics unit or course.)

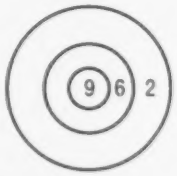
(The 1988 NCTM Yearbook, ***The Ideas of Algebra, K-12,*** is a good source for discussion of what is appropriate. It contains both philosophical articles and activity suggestions. The "Which Does Not Belong?" activity is presented on page 221 of the yearbook.)

REVERSALS

Reversal activities can be introduced and used at almost any point and at any level. These activities are opportunities to "turn the tables" and require the student to construct a situation. For example, in "Which Does Not Belong?" the stem usually states the condition with one "oddball" appearing in a list that states the condition. In some situations the process of elimination operates without the student demonstrating that he or she really understands what the condition means. Reversals allow students to construct the foils by applying their knowledge of the concept. An example of a reversal comes from the familiar "dart board" problem:

"What is the least number of darts needed to score 34 on the dart board?"

While the answer is 6, additional questions could be addressed.



A reversal of the problem could be: "What is the range of possible scores, knowing the number of darts it takes to solve the problem?" Another reversal would be "Which scores are not possible, using the number of darts that solve the original problem?" Reversals provide the opportunity to "turn it up a notch" by increasing the level of mathematical investigation. A reversal activity of this type negates "fun and games" or guessing because the student must be accountable for results.

The reversal of "Which Does Not Belong?" follows:

1. Make a list of four numbers, three of which are perfect squares and the fourth not a perfect square. 4, 16, 25; 30
2. List the coordinates of four points, three in the second quadrant and one in another quadrant. $(^{-}2, 5)$, $(^{-}5, 2)$, $(^{-}3, 14)$; $(3, ^{-}5)$

Reversals such as these are not intended for developmental work because one cannot teach a topic using this approach; the intent is to verify student knowledge, to extend an idea, or to solidify student understanding of a concept.

3. Write four expressions, three of which are factors of $6x^2y$ while the fourth is not.

   $6x$, $6xy$, $2y$; $3xy^2$

4. Write three equations equivalent to $6x = 10$ and one equation that is not.

   $3x = 5$, $6x + 19 = 29$, $12x = 20$; $6x + 5 = 5$

5. Write four radicals where three are between 6 and 7 and the fourth is not.

   $\sqrt{40}$, $\sqrt{42}$, $\sqrt{45}$; $\sqrt{6.5}$

6. Write three expressions equivalent to $8a$ and one that is not.

   $5a + 3a$, $11a - 3a$, $a + 7a$; $2a(4a)$

7. Consider systems of equations. Write the equations for three lines that are parallel and a fourth that is not parallel to the other three.

   $y = 2x + 5$, $y = 2x + 10$, $y - 2x = 7$; $y = 4x + 5$

FRONT LOADING

There are independent topics that, taught at various stages within the algebra course, can be addressed during the first few weeks of the school year. This process of "front loading" the curriculum affords the teacher the opportunity to include a number of key areas that then can be reviewed at many different times during the year. Most importantly, the preview will "grease the skids" when the topic is addressed formally in the algebra course. Examples of these topics in a first-year algebra course might include:

- coordinate system
- Pythagorean theorem
- simplifying radicals
- basic geometric concepts (a variety of concepts that will appear during the algebra course)
- simple probability
- statistical measures: mean, median, mode

For the most part these topics should have been included in grades 7 and 8. This recommendation does not call for a complete development of a unit of study for each topic, but rather a brief re-exposure to allow maintenance of these skills and concepts during the entire school year. Formal algebraic presentation will not have to be at "square one." This would help bridge the gap between the seventh/eighth-grade treatment and the algebraic development later in the course.

The following article, "Algebraic Anticipation," discusses front loading and provides examples for presentation. The reference to preview new material noted in the section "Which Does Not Belong?" includes an exercise using slope. This is an example of a front-loading activity.

"Follow the Leader" deals with algebraic expressions and the use of algebraic symbols —an excellent preview or front-loading activity that is easy for students.

ALGEBRAIC ANTICIPATION

David J. Glatzer, West Orange Public Schools, West Orange, New Jersey. ***The New Jersey Mathematics Teacher,*** Vol. 43, No. 1, Fall, 1985.

The quadratic formula represents one topic within the algebra curriculum where students need an introduction to the new notation long before formal work with the formula. In our earlier units on radicals and signed numbers, do we provide exercises which are designed to anticipate the format employed in answers coming from the quadratic formula?

Consider the solution to the equation: $x^2 + 6x + 4 = 0$

$$x = \frac{-6 \pm \sqrt{36x - 4(1)(4)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{20}}{2} = \frac{-6 \pm \sqrt{5}}{2} = -3 \pm \sqrt{5}$$

FRONT LOADING

The purpose of this article is to propose that students would be better prepared to deal with the type of notation resulting from the quadratic formula if they had a sequence of experiences involving components of the notation.

In the unit on radicals, students perform operations involving radicals and simplify expressions containing radicals. Typically, students are not asked to do problems that "resemble" the quadratic formula notation. Why not offer the following during the bulk of an elementary algebra course?

Simplify:

1. $4 \pm \sqrt{9}$
2. $4 \pm \sqrt{36}$
3. $\frac{\pm \sqrt{81}}{3}$
4. $-9 \pm \sqrt{36}$
5. $-9 \pm \sqrt{81}$
6. $-9 \pm \sqrt{0}$
7. $\frac{-8 \pm \sqrt{16}}{2}$
8. $-4 \pm \sqrt{9}$
9. $4 \pm \sqrt{8}$
10. $-4 \pm \sqrt{8}$
11. $\frac{4 \pm \sqrt{8}}{2}$
12. $\frac{-4 \pm \sqrt{8}}{2}$

In earlier units dealing with signed numbers and algebraic manipulation, the following exercises can be included:

Simplify:

1. ± 8
2. 3 ± 8
3. -3 ± 8
4. $3 \pm 1/2(8)$
5. $\pm 8/2$
6. -8 ± 8
7. 0 ± 8
8. $1/2 \pm 8$
9. $6a \pm 8a$
10. $-4a \pm 8a$

Suggested exercises in this article offer a way for teachers to prepare students for a new type of notation. Although this article *was* stimulated by a specific piece of content from the algebra curriculum, the author contends that the idea of "ANTICIPATION" should be an integral part of long-range planning in mathematics instruction.

FOLLOW THE LEADER

Worksheet

	(1)	(2)	(3)	(4)	(5)	(6)
x	8	10	3	$^{-}5$		
y	5	7	$^{-}3$	0	5	
$x + y$	13				12	12
$x - y$	3					6
xy	40					
$2x + y$	$2(8) + 5$ 21					
$x + 2y$	$8 + 2(5)$ 18					
$x - 2y$	$8 - 2(5)$ $8 - 10$ -2					
$x^2 + y$	$8^2 + 5$ $64 + 5$ 69					
$x^2 - y$	$8^2 + 5^2$ $64 + 5^{-25}$ 39					
$y^2 - x^2$	$5^2 - 8^2$ $25 - 64$ $^{-}39$					
$^{-}(x + y)$	$^{-}(8 + 5)$ $^{-}13$					
$^{-}x + y$	$^{-}(8) + 5$ $^{-}8 + 5$ $^{-}3$					

WHO'S ON FIRST

This activity is designed to address the issue "I never know where to start." Certain activities are independent of content and can be used at any time. This activity, for example, can be used many different times throughout the school year. The student is to concentrate on determining the first step needed in the process of solving standard problems; this is not a developmental activity. Also, it is not recommended that students solve problems completely. Problems can be solved at a later time; the intent here is to dramatize the importance of concentrating on the first step.

Approach: Given a series of independent problems, the student will indicate the step that he or she will use to start. The student also should be asked to discuss the rationale behind the selection of the first step.

Example: From \$50.00, you expect to buy a tape of a movie for \$19.95 and as many blank tapes as possible for \$3.95 per tape. How many blank tapes will you be able to buy?

One should contrast the formal versus the informal approach to the problem. In the formal setting, one would set up an inequality: $19.95 + 3.95x < 50.00$. Algebra students might, however, benefit more from an informal approach such as, "Let's find out how much money we have available for tapes." Then the first step might be to subtract (50.00 – 19.95). After some discussion, the situation could be formalized into algebraic language by setting up the inequality.

Comments: As a rule, it is good to pick examples where there is a clear-cut first step. It is valuable also to see student responses when there is an option of which step should be used first.

The focus of this activity is to concentrate on the first step. Complete solution of the problems should be done at a later time so that the intent of this exercise is realized.

The activity should begin with an application problem so the student is comfortable with the context and has a feeling of practicability. Also, it should not be too easy, so the student says "What's the use?" A relatively clear-cut first step will help prevent guessing from becoming the usual procedure. This is a "do not solve" activity; therefore, attention should be on processes, options for beginning, the idea that there are no absolutes for solution, and use of analogies when approaching the solution.

WHO'S ON FIRST

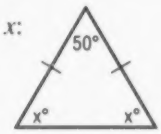
Directions: For each of the following indicate what your first step would be. Do not solve the problem.

1. Solve for x:

   $3(x + 5) - 4x = 2(x - 4)$

2. Find the average of the scores: 82, 76, and 90.

3. Find x:



4. Simplify: $\sqrt{50}$

5. Find the least common multiple of 12 and 18.

6. Evaluate $5a^2b^3$ when $a = 2$ and $b = -3$.

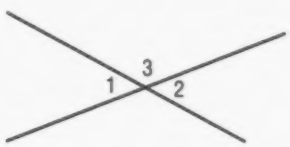
7. Solve for x: $\dfrac{3}{7} = \dfrac{x}{42}$

8. You have four scores of 82, 78, 90, and 70. What score do you need to get on the fifth test in order to obtain an average score of 81 for the five tests?

9. From \$50.00, you expect to buy a tape of a movie for \$19.95 and as many blank tapes as you can buy at \$3.95 per tape. How many blank tapes are you going to buy?

10. Given: $m\angle 1 = 5x^\circ$
    $m\angle 2 = (2x + 51)^\circ$

    Find: $m\angle 3$



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APPLICATIONS

The activity "Who's on First?" leads into a discussion of applications. Students should get frequent practice using their problem-solving skills in an application setting. Drawings and diagrams often are useful in problem solution. This section is not intended to exhaust the types of application problems that can be used with an algebra class. Many applications do exist. They should be used to illustrate the power of algebra and to help students gain facility and confidence in their thinking about problems using algebra. Some examples follow.

1. Water Witch Well Drillers charge their customers $350.00 to come to the well site and $20.00 per foot to drill a well.

   a. Using x for the number of feet, write an expression for the cost of drilling a well x feet deep.

   b. How much would you pay for a well that is: 50 feet deep? 80 feet deep? 125 feet deep?

   c. If a person was charged $2,150.00 for a well, how deep is the well?

2. Taka-Taxi Co. charges $1.35 immediately upon entering the taxi, and $1.80 for each mile driven.

   a. If x is the number of miles from the airport to your hotel, write an expression of the cost for the taxi ride.

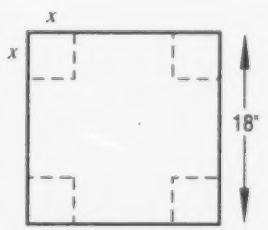
   b. Write an equation representing the total cost of the taxi ride.

   c. Find the cost if the distance to your hotel is 5 miles; 9 miles.

   d. How many miles is your hotel from the airport if the cost of the ride was $6.75? If the cost was $26.55?

   e. How far would the taxi take you for $20.00?

3. Find the maximum volume of an open box made from a square sheet of cardboard that is 18 inches on a side.



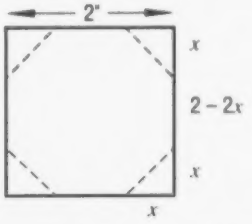
How is volume calculated?

How is each dimension represented?

$(18 - x)(18 - x)(x)$ → max vol

If the problem is presented early in the course, do not solve it; rather, discuss how one might approach the situation. If a transparency is used, it could be saved until the student's background is sufficient to solve the problem. The point is that algebra is the stepping stone to the solution.

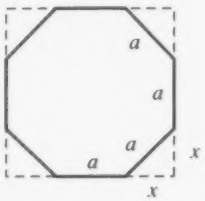
APPLICATIONS

4. 

The corners of a 2" square are cut off to form a regular octagon. How much should be cut off each side?

Use geometry to get started, then bring in algebra to provide the power to solve the problem.

Since the octagon is regular, each side is $2 - 2x$, allowing use of the cut-off right triangles corners of the square to solve for x.



$a^2 = x^2 + x^2$

$(2 - 2x)^2 = x^2 + x^2$

$(2 - 2x)^2 = 2x^2$

$2 - 2x = x\sqrt{2}$

This will be solved later in the course (keep the transparency).

One solution: $2 - 2x = x\sqrt{2}$

$2 = x\sqrt{2} + 2x$

$2 = x(\sqrt{2} + 2)$

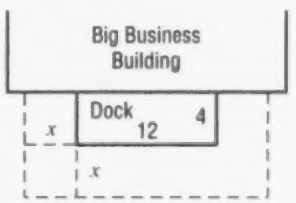
$$\frac{2}{\sqrt{2} + 2} = x$$

$x \approx > 0.59$ using knowledge of $\sqrt{2}$ or a calculator

5. The loading dock for Big Business is too small. The plan is to double its size.

It is to be increased the same amount, both length and width.

If the original dimensions are as shown, what are the new length and width?



Area: (4)(12) = 48

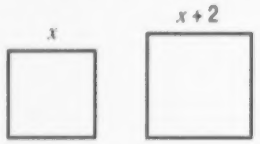
Begin with drawings and move to algebraic representation.
(Fifth graders can explain this problem using graph paper.)

The new width is $4 + x$, and the new length is $12 + 2x$.

The new area is represented by $(4 + x)(12 + 2x) = 96$

$x = -10, 2$ Using "reasonable answers," an increase of 2 is required.

6. Dimensions of two squares differ by 1, and their areas differ by 19. What are the lengths of the sides?



The areas are x^2 and $(x + 1)^2$,
so $(x + 1)^2 - x^2 = 19$
$x = 9$ then the sides of the squares are 9 and 10.

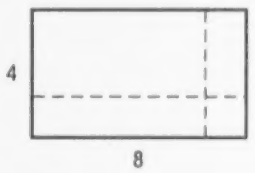
Many application problems are found in source books, recreational books, and textbooks. (One in particular, by Foerster, contains many interesting problems and is listed in the reference section. Applications provide opportunities for review and maintenance, for solidification of concepts, and for preview. One finds, through class discussion, that there are many things students *do* know. Algebra allows confirmation of what one does in one's head, and allows a student to determine whether all possible answers are accounted for. For example:

7. What numbers are equal to their squares?

$$x = x^2$$
$$x^2 - x = 0$$
$$x(x - 1) = 0$$
$$x = 0, 1$$

Note: One common error involves dividing by x, thus losing one of the solutions.

8. The same amount is to be subtracted from the length and width of the rectangle so the resulting area is one-half the original area. How much must be subtracted?
Original area: $(4)(8) = 32$
New area: $(4 - x)(8 - x) = 16$
Solving the quadratic equation,
$x = 10.47, 1.53$



Reviewing the problem indicates that 10.47 is not a possible solution, and only one solution satisfies conditions of the problem. Checking the possibility and reasonableness of obtained "solutions" is an important aspect of using algebra.

CONCEPT DEVELOPMENT

Concept development can and should occur in a variety of ways by:

- expressing ideas mathematically
- comparing and contrasting new concepts with those already known
- use of vocabulary
- stating relationships
- looking for and showing patterns
- communicating mathematical ideas both in written and in oral form

Concept development in this discussion is not theoretical in the sense of learning theory or the psychological aspects of learning. Rather, it refers to those ideas that should be kept in mind and used, as new concepts are introduced and expanded upon.

At the Popular Pizza Place, toppings one might order are pepperoni, onion, sausage, mushrooms, or green peppers. How many different pizza combinations can one order (from plain to "the works")?

This is a counting problem. The student could order any one of 32 pizzas. A variety of ways to show the problem solution can be used, from diagrams and lists to the eventual rule.

One could list all the ways to get pizza with no toppings, 1 topping, 2 toppings, and so on. The count would be 32.

One could consider:
1 way to get plain, which means 5 toppings are left off (this also shows that there is 1 way to get "the works").

For 1 topping there are 5 possibilities. Each time there are 4 toppings left off the pizza, so there are 5 ways to get 4 toppings. Similarly, there are 10 possibilities for 2 toppings and for 3 toppings.

Pascal's triangle could be used to show the solution using the 5th row.

There are two choices (yes or no) for each of the 5 toppings, which results in $2 \times 2 \times 2 \times 2 \times 2$, or $2^5 = 32$ possible pizzas.

This problem incorporates many components of concept development.

SUM AND PRODUCT

Intent: To preview the concepts of factoring special binomials and trinomials.

Approach: Students are given two numbers and are asked to find a number pair whose sum is the left number and whose product is the right number. The two boxed numbers can appear on card stock, transparencies, or a worksheet.

Examples:

5	6

13	36

2	$^-15$

0	$^-25$

0.3	0.02

Comments: Students do not have to be told that they will use thinking developed in this activity as part of factoring algebraic expressions. This can come later (when review examples from this handout can be used). Additional examples from factoring polynomials are:

1.

0	$^-100$

number pair: 10, $^-10$

$x^2 + 0x - 100$
$x^2 - 100$
$(x + 10)(x - 10)$

2.

$^-4$	4

number pair: $^-2$, $^-2$

$x^2 - 4x + 4$
$(x - 2)(x - 2)$

3.

$^-5$	6

number pair: $^-2$, $^-3$

$x^2 - 5x + 6$
$(x - 2)(x - 3)$

4. Find the factors for:

$^-5$	6

$\rightarrow$

$x^2 - 5x + 6$
The product is 6 and the sum is $^-5$.
$(x \quad)(x \quad)$

Since the number pair for the product and sum (the boxed numbers) is $^-2$ and $^-3$, we get $(x - 2)(x - 3)$ as factors.

5. Find the factors for $x^2 + 13x + 36$.

The product is 36 and the sum is 13,
so the number pair is 9 and 4.

SUM AND PRODUCT

+	×

(1) 5	6	(9) .2	.01
(2) 13	36	(10) 0	$^{-}25$
(3) 11	18	(11)	
(4) 2	$^{-}15$	(12)	
(5) $\frac{7}{12}$	$\frac{1}{7}$	(13)	
(6) $\frac{5}{8}$	$\frac{6}{8}$ or $\frac{3}{4}$	(14)	
(7) .05	.0006	(15)	
(8) $^{-}8$	15	(16)	

HOW DO YOU KNOW THAT?

Intent: Students focus on fundamental concepts and communicate ideas about those concepts.

Approach: Students are asked to write one or two sentences to address each question. For some situations a diagram accompanying the statement is appropriate.

Examples:

1. How Do You Know That (HDYKT): $4 \times 5\ 1/2 \neq 20\ 1/2$
   Possible Response: 5 1/2 is 5 + 1/2; therefore, the 4 has to multiply both the 5 and the 1/2.

2. HDYKT: $4^{15} > 2^{29}$

3. HDYKT: A triangle cannot have two obtuse angles.

Comments: There will be considerable variation in student response for this type of activity. Look for and emphasize conceptual understanding rather than algorithmic procedures. It is beneficial for students to work on these questions in pairs or in cooperative learning groups. An example of a major concept question is:

HDYKT: $(5 + 4)^2 \neq 5^2 + 4^2$?

(This is an example where a geometric representation would be helpful.) This example easily could be generalized to $(a + b)^2 \neq a^2 + b^2$. It is important to review such major concepts periodically to keep the ideas fresh in the students' minds.

Some of these problems can be added to assignments at various times throughout the year. Possibly assign fewer exercises and present some HDYKT questions. Such questions also could be added to quizzes and tests.

HOW DO YOU KNOW THAT?

Analogies

The use of analogies lends itself to the verification of concept development. Some examples are:

$x + y$: Binomial :: $a + b + c$: (**Trinomial**)

$\sqrt{\ }$: Square Root :: ($\sqrt[3]{\ }$) : Cube Root

(3, 4) : 1st Quadrant :: (-5, 2) : (**2nd Quadrant**)

$-x$: Opposite :: $\left(\frac{1}{x}\right)$: Reciprocal

Making Lists

Making lists is another form of concept verification.

- List 10 perfect squares.
- List four formulas.
- List _____ binomials that are the difference of two squares.
- List _____ equations whose lines are parallel to $2x + 3y = 6$.
- List _____ equations of parabolas that open downward.
- List 15 words that have to do with circles.

HOW DO YOU KNOW THAT?

1. How do you know that the following product is positive?

$$(^{-}2.8)(^{-}3.9)(^{-}4)(^{-}5.3)$$

2. How do you know that $3(x + 5)$ is not equal to $3x + 5$?

3. How do you know that $\sqrt{39}$ is between 6 and 7?

4. How do you know that $5a + 7b - 13c$ cannot be simplified?

5. How do you know that $x = 5$ is not a solution to the following equation?

$$3x + 12 = 0$$

6. How do you know that the solution to the equation $3x + 12 = 0$ must be a negative number?

7. How do you know that the following sums are all positive?

$^{-}6 + (^{+}11)$ $\quad$ $^{+}90 + (^{-}85)$ $\quad$ $^{-}7000 + (7000) + (^{+}5)$

23

CONSIDERATIONS FOR EQUATIONS

1. To promote equation solving, it is helpful to use a problem where one wants to find a mystery number. The most effective method is to use an equation for which the mystery number is not obvious.

   Example: I am thinking of a number that if I add 3 to twice the number, I get the same result as subtracting 1 from 5 times the number. Find the mystery number.

   $$2x + 3 = 5x - 1$$
   $$4 = 3x$$
   $$1\ 1/3 = x$$

2. As above, some equations with fractional answers should be included. Students should not view fractional answers as undesirable or unusual.

3. For certain applications, students need to be aware that answers must be related to the situation's physical nature. For example: In solving an equation to obtain the number of sides in a polygon, it would not be acceptable to have a solution of 7 2/3. This type of solution should cause the student to go back and double-check his or her work.

4. Do not postpone the consideration of literal equations. The equation $ax + b = c$ should be considered during the study of standard equations such as $5x + 4 = 20$.

5. Although the first major study of equations will be confined to linear equations, students should see a variety of other equations in order to know where the study of equations is headed.

   Examples:

   1) $x = 9$

   2) $\sqrt{x + 4} = 5$

   3) $2^x = 32$

   4) $\dfrac{1}{x - 2} = \dfrac{1}{7}$

These equations may be solved by inspection. However, the immediate goal is not to solve them but to expose students to a variety of equations.

6. Stress the importance of checking solutions to equations. Use the check as a way of reinforcing basic skills covered at earlier stages of the course. It is appropriate to make the "check" a stand-alone activity. (Refer to the activity "The All-Important Check.")

7. Require students to communicate an understanding of what equation solving really means.

   Example: How do you know that $x = 5$ is not a solution to the equation $3x + 12 = 0$?

   Example: $x = 5$ and $x = 4$ are solutions to the quadratic equation $x^2 - 9x + 20 = 0$.

   What can you conclude about $x = 7$ in relation to the same quadratic equation?

8. Give students experiences with conditional equations where one number works; two numbers work; all numbers work; no number works:

   $3(2 + 4x) = 16$ $\quad x^2 - 5x + 6 = 0$ $\quad 3(x + 2) = 3x + 6$

   $3(x + 2) = 3x + 5$

9. Make a natural extension of equation solving, such as $(x - 2)(x - 3) = 0$ extended to $(x - 2)(x + 3)(x - 5)(x - 7) = 0$.

10. Use equations to represent diagrams.
    (See page 29, "Working with Equations.")

EXPRESSIONS AND EQUATIONS

Foerster (1990) refers to the three major operations performed in algebra. They are—

- writing expressions representing something in the real world;
- finding the value of an expression when one knows what x equals;
- finding the value(s) of x when one knows what the expression equals.

For example:

- Write an expression for the perimeter of an equilateral triangle.
- Write an expression for (and find) the perimeter if a side of an equilateral triangle is 5.
- Write an equation for the perimeter of an equilateral triangle if the perimeter is 48. Solve to find the length of a side.

Students should see a clear distinction between the meaning of an expression and the meaning of an equation. This includes the distinction between simplify and solve. Solving an equation produces a solution (often a number), whereas simplifying an expression produces another expression.

$$3(5x - 2) = 37 \rightarrow x = 2.87$$

$$3x + 5 - 2x + 7 + 8x \rightarrow 9x + 12$$

Communication: An important aspect in the process of understanding algebra is to be able to communicate both orally and in writing.

1. Read aloud: $3a + 2b$, $2x - 3y = 7$

   $5 \leq a \leq 9$, $\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$

2. Write five ways to express the product of a, b, and c.

   abc, $a(bc)$, $(ab)c$, cba, $(a)(b)(c)$

3. Write an algebraic expression containing parentheses and multiplication and subtraction operations.

   $5(x - y)$

4. Write $2x^4$ in another way using addition.

   $2x^4 = x^4 + x^4$

Combining Terms: Using a sequence such as the following is helpful:

$12x + 5x = 17x$

$12x + 6x = 18x$

$12x + 7x =$

• • •
• • •
• • •

$12x + 5x + 8x =$

$12x + 5x + 8 =$

• • •
• • •
• • •

$12x + 5x - 9x =$

While students are evaluating expressions, they should learn to use parentheses appropriately.

Evaluate: $3x^2y + 4xy$ when $x = 2$, $y = 5$

$3(2)^2(5) + 4(2)(5)$

$3(4)(5) + 4(2)(5)$

$60 + 40$

100

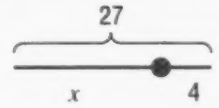
WORKING WITH EQUATIONS

One way to begin equation work is to use the "mystery number" approach mentioned earlier. Algebraic procedures are not applied at first, but rather a guess-and-test method of solution is allowed. Students are not held responsible for algebraic steps at this point; organized solutions along algebraic lines will come later.

One possible activity is to provide the equation with a solution and have students check to see whether the solution is correct. Checking should be emphasized. Checking is an integral part of the equation-solving process. Checking is a confirmation of all the students' previous work with expressions, parentheses, and operations. Situations can be presented where there are multiple solutions, and other situations where only one number works, two numbers work, or all numbers work. "The All-Important Check" is one format for this activity.

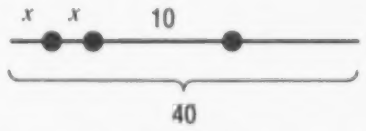
The use of diagrams is another approach to the development of writing equations or inequalities.

1. Write an equation for the diagram.



$x + 4 = 27$

2. Write an inequality for the diagram.



$2x + 10 < 40$

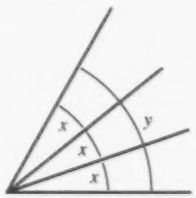
Use of diagrams is also a good review activity. See page 30, "Equations from Diagrams."

Expressing y in terms of x is another possible activity. This leads to equation formulation and can be introduced via geometric concepts.

$y = x$

$y = 90 - x$

$y = 3x$



The problems on pages 31 and 32 deal with expressing y in terms of other variables. These pages provide the type of practice needed and offer a connection with geometric concepts learned in previous grades.

The All-Important Check

Purpose: To focus attention on the formal process of checking a potential solution to an equation. Also, to reinforce the concept of a solution to an equation.

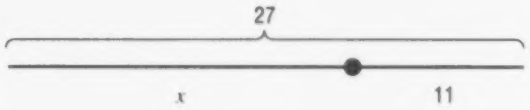
Approach: Use a worksheet format similar to the sample below. Have students show the actual checking process for given equations and potential solutions. Potential solutions may or may not be true solutions.

Equation	Potential Solution	Check	Response
1. $3x + 5 = 26$	$x = 6$	$3x + 5 = 26$ $3(6) + 5 \stackrel{?}{=} 26$ $18 + 5 \stackrel{?}{=} 26$ $23 \neq 26$	6 is not a solution
2. $6x - 4 = 40$	$x = 8$		
3. $\frac{5x}{3} = 60$	$x = 36$		
4. $7(x + 3) = 0$	$x = -3$		
5. $\frac{x}{2} + \frac{1}{4} = \frac{3}{8}$	$x = 2$		

Note: The question mark in the "check" column should be used in *each* line until the check is completed with a true or false statement.

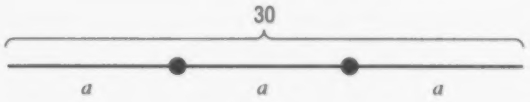
Equations from Diagrams

(1)

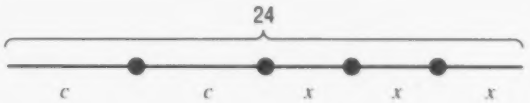


$x + 11 = 27$

(2)

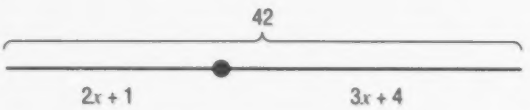


(3)

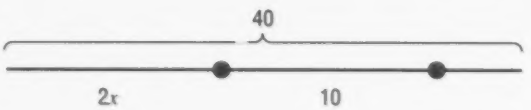


(4)

(5)



(6)

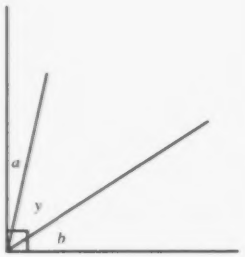


Express y in Terms of a and b

SHEET A

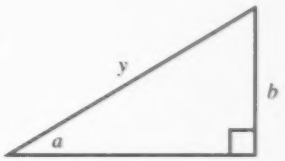
(1)

(2)



(3)

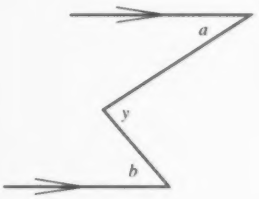
(4)



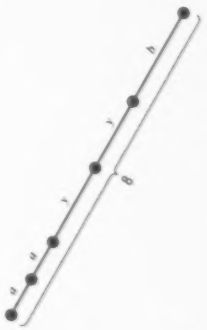
(5)

a, y, b are consecutive whole numbers

(6)



(7)

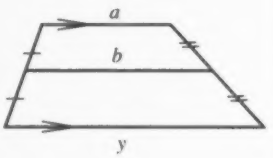


(8)

a is the average of b scores and y is the sum of b scores.

(9)

trapezoid



Express y in Terms of x
SHEET B

1. $2y = x$
   $y = 1/2x$

2. $\frac{1}{5}y = x$

3. $3y + 5 = x$

4. $3y + 5 = x + 5$

5. $3y + 5 = x - 5$

6. x = 50% of y

7. x = 100% of y

8. x = 10% of y

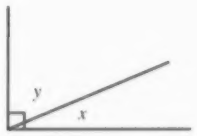
9. $\frac{x}{y} = \frac{1}{2}$

10. $\frac{x}{y} = \frac{6}{7}$

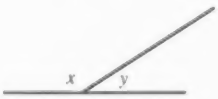
11. $x = y^2$ $(x \geq 0)$

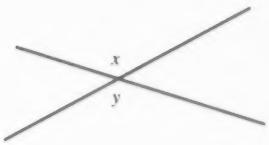
12. $x = y^2$ $(x \geq 0, y \geq 0)$

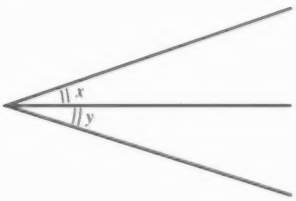
13. $x = y^3$

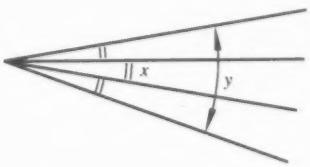
14. 

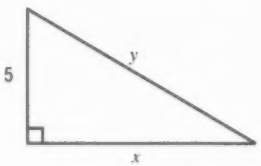
$x + y = 90^\circ$
$\therefore y = 90^\circ - x$

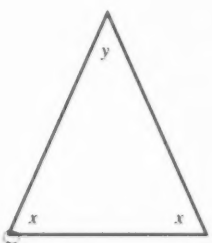
15. 

16. 

17. 

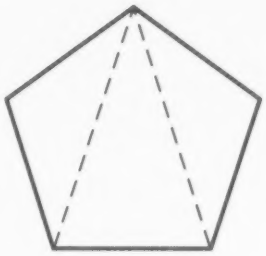
18. 

19. 

20. 

DEVELOP A FORMULA

1. Write a formula for the sum of the measures of the interior angles of a convex polygon of N sides. (An added benefit to this problem is a discovery experience for students as they create the necessary expression.)



A discussion:
For what figure do you know the sum of the measures of the interior angles? (Triangle)
Can this information be used in our figure? (Yes)
Draw diagonals to make triangles. How many? (3)
Now find the number of degrees in the interior angle of the pentagon. (540)

Set up a table to discover a pattern leading to the formula.

Number of Sides	Number of Triangles	Sum of Angle Measures
3	1	180
4	2	2(180) = 360
5	3	3(180) = 540
6	4	4(180) = 720
•	•	• •
•	•	• •
•	•	• •
9	7	7(180) = 1260
•	•	• •
•	•	• •
•	•	• •
N	$N - 2$	$(N - 2)$ 180

The formula is: $S_I = (N - 2)\ (180)$

2. Reversal:

If the sum of the interior angles is 3240, how many sides does the polygon have?

Note: Before the student starts this problem, he or she should be aware that the solution must be a positive whole number greater than or equal to 3.

$S_I = 180(N - 2)$ Write the formula! $S_I = 180(N - 2)$

$3240 = 180(N - 2)$ You can divide or distribute $3240 = 180(N - 2)$

$3240 = 180N - 360$

$\frac{3240}{180} = N - 2$ Algebraic steps

$18 = N - 2$ $3600 = 180N$

$20 = N$ $20 = N$

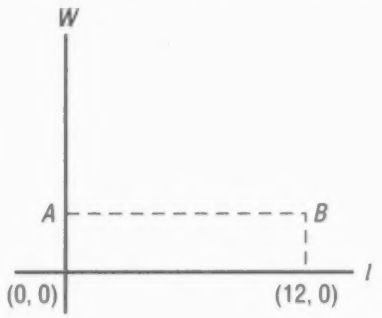
33

DEVELOP A FORMULA

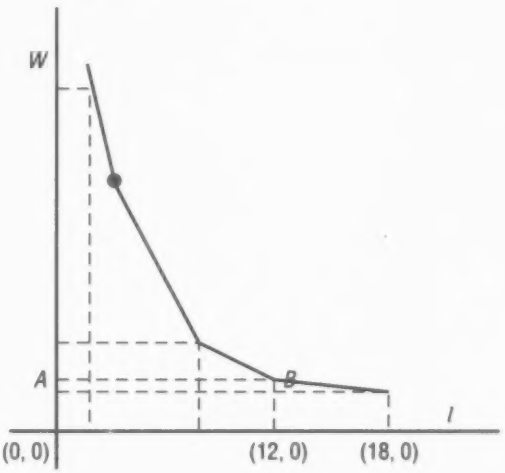
3. Find variables in other formulas:

Find A in $A = 1/2\ h(b_1 + b_2)$	Also find b_1.
Find $z = 4a + 3(b + c) - g$	Find any letter. Finding z is not using algebra; finding a, b, c, and g is using algebra.

4. Using graphs to solve problems.
   a. Find A and B so that the area of the rectangle is 36.



   b. Consider the graph.



Choosing points along the graph going from left to right, as one coordinate increases, the other coordinate decreases.

The function is decreasing.

If you consider the rectangles in the figure, one dimension increases as the other decreases.

When the rectangle is a square, find the perimeter and area.

What are the perimeter and area when the length is twice the width?

5. Baseball Ticket Problem (adapted from Foerster, *Algebra I*)
Reserved seats cost \$3 and general admission tickets cost \$2.
Write an expression for the total number of dollars for tickets sold:

$$3r + 2g$$

How much money is received for selling:

100 reserved	0 general admission	\$ _____	Essentially this is arithmetic.
0 reserved	400 general admission	\$ _____	
100 reserved	100 general admission	\$ _____	

Write an equation representing each of the above.

If you must sell \$30 in tickets, write an equation to show the condition:

$$3r + 2g = 30$$

Students should realize that they cannot sell fractional numbers of tickets. Only the ordered pairs with integer values are solutions of the equation.

If you sold 0 reserved tickets, how many general admission tickets would you have sold? (15) If you sold 0 general admission tickets, how many reserved tickets would you have sold? (10)

On the grid, plot the points representing 0 reserved tickets sold and 0 general admission tickets sold. (15, 0) and (0, 10)

Draw the segment connecting these points.

Find a point on the line. See whether the coordinates represent a possible combination of tickets sold.

Follow along the line and record all ordered pairs that could represent tickets sold. Are these all the combinations of tickets that could be sold in order to obtain \$30?

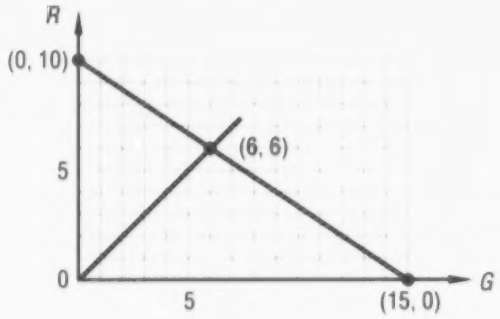
Would it be possible to sell the same number of reserved and general admission tickets?

If you think so, plot the coordinates to see whether the point falls on the line.

Choose other pairs of numbers representing a sale of the same number of reserved and general admission tickets (not restricted to $30 in sales.) Plot these points. Note that the points fall in a straight line, and that $r = g$.

$r = g$ is an equation!

Draw the line for the equation.



These two equations form a system.

$$\begin{cases} 3r + 2g = 30 \\ r = g \end{cases}$$

The intersection of their graphs is a point whose coordinates satisfy both equations.

This is a preview of the future for students.

Note: Such detailed analysis and discussion of a problem or application is important and rare.

SUGGESTIONS FOR ORGANIZING STUDENT WORK

For students to be successful, it is important that their work in algebra be organized in a reasonable manner. Teachers need to hold students accountable for a high degree of organization.

1. Evaluate $3a^2$ if $a = 5$.

   Solution: $3(5^2)$
   Stress the importance of parentheses in evaluating expressions.

2. Proper use of equal signs and fraction symbols.

   $\frac{6}{x}$ (= is needed) $\frac{5}{32}$ $\quad \frac{x}{8} = \frac{3}{6} = 4$

   Improper use resulting in incorrect statement; need to show solution steps.

3. Proportions:

   $$\frac{18}{5x} = \frac{81}{100}$$

   Set up ratios in proper form, and organize the work in an orderly way. Lining up = signs helps.

   $$18 \cdot 100 = 81 \cdot 5x$$
   $$81 \cdot 5x = 18 \cdot 100$$
   $$5x = \frac{18 \cdot 100}{81}$$
   $$x = \frac{18 \cdot 100}{81 \cdot 5}$$

4. Solving quadratic equations by factoring.

   $x^2 - 5x + 6 = 0$
   $(x - 2)(x - 3) \leftarrow$ (needs = 0)
   $x = 2, x = 3$

   The equation must be maintained for the concept of setting factors equal to zero to apply.

5. Systems of equations.

   (a) $3x + 2y = 4$
   (b) $4x - 3y = 8$

   $3x$(a) $9x + 6y = 12$
   $2x$(b) $8x - 6y = 16$

   Students need a way of organizing work. Use some labels — e.g., (a), (b), as a device to keep track of work.

6. Simplify radicals.

   $\sqrt{98} + \sqrt{50} + \sqrt{32} = ?$

   $\sqrt{98} = 7\sqrt{2}$
   $\sqrt{50} = 5\sqrt{2}$
   $\sqrt{32} = 4\sqrt{2}$
   $16\sqrt{2}$

   Sometimes it is more desirable to do a problem vertically rather than horizontally.

REVIEW ACTIVITIES

Several review activity ideas are provided in the next few pages. Of course, many ideas previously presented in this document also can be developed as review activities. Review should focus on the major topics of algebra, especially those areas that students tend to forget or in which they have a weakness. Review can be made to be fun so that algebra continues to be an enjoyable experience.

The "Who Has?" activity is in the style of a treasure hunt. The entire class participates. It can be used with different cards to cover many concepts in the algebra course. Students also can generate this activity themselves. The scope of topics for a particular game may be kept small. However, it is possible to use the activity as a major review with many topics included.

An activity pertaining to vocabulary is "Algebra from A to Z." Students (at their seats individually, or with the teacher using an overhead projector) are to list algebraic words beginning with each letter of the alphabet. For example, A = axis, B = binomial, C = coefficient, and so on. An interesting part of this activity is to see how many related words may be generated by naming a particular work (for example, *graphing, line, slope, coordinate*).

In implementing the "Who Has?" activity, teachers can use a set of cards (5" × 8"), with each student getting one card. In turn, the cards are used as individuals respond and pay attention to the clues. After using the card format once, students may use the worksheet in a small-group setting. This way, each student or group is responsible for the entire collection of items.

Below is a sample of a typical "Who Has?" card.

90
I HAVE 90.
WHO HAS THIS
DIVIDED BY THE
LARGEST ONE-DIGIT
PERFECT SQUARE?
06

Who Has . . .?

WHO HAS 8?

a. I have 90. Who has this divided by the largest one-digit perfect square?

b. I have 8. Who has my number plus the solution to $3x - 15$?

c. I have 4.5. Who has my number times the solution to $x - 10 = 0$?

d. I have 11. Who has the complement of this number of degrees?

e. I have 5. Who has the reciprocal of my number?

f. I have 13. Who has this minus the number of prime factors of 12?

g. I have 81. Who has the square root of 81?

h. I have 0. Who has my number plus $10^3 + 10^2 + 10^1 + 10^0$?

i. I have 2. Who has my number to the fifth power?

j. I have 1111. Is this a good place to stop? Wasn't this fun?

k. I have 10. Who has my number plus the value of r^3 when $r = 2$?

l. I have 45. Who has 50 less than my number?

m. I have 32. Who has my number minus the number of ounces in one quart?

n. I have -5. Who has the absolute value of my number?

o. I have 1/5. Who has 10 times my number?

p. I have 79. Who has this plus $\sqrt{121}$?

q. I have 9. Who has 50% of my number?

r. I have 18. Who has my number with its digits reversed?

REVIEW ACTIVITIES

Where Did I Go Wrong?

Purpose: To reinforce use of the distributive property.

Approach: This activity involves the discussion of common errors associated with the use of the distributive property. With examples on the chalkboard or transparency, small groups of students are to arrive at consensus concerning the nature of the error made. **Use examples that show only incorrect responses.** Have students explain the reason for each error. This will require considerable thinking in each illustration.

Examples:

1. Simplify: $2(2a + 5) =$
   A. $4a$
   B. $4a + 5$
   C. $4a + 7$
   D. $2a + 10$

2. Simplify: $3(5a - 1) =$
   A. $15a - 1$
   B. $15a + 3$
   C. $15a + 2$
   D. $15a - 2$

3. Simplify: $^{-}4(2a - 7) =$
   A. $^{-}8a - 7$
   B. $^{-}8a - 28$
   C. $8a + 7$
   D. $8a + 28$

4. Simplify: $3(2a + b + 4) =$
   A. $6a + 3b + 4$
   B. $6a + b + 12$
   C. $6a + b + 4$
   D. $6a + 3b$

Cross-Number Puzzle

Purpose: To reinforce the process of evaluating expressions containing one or two variables.

Materials: A blank cross-number puzzle designed to match a set of clues.

Approach: Explain to students how a cross-number puzzle works. Explain that the following values must be used in the clues in order to arrive at a correct answer. Once they have evaluated the expression correctly, they should write the answer in the corresponding blanks on the puzzle.

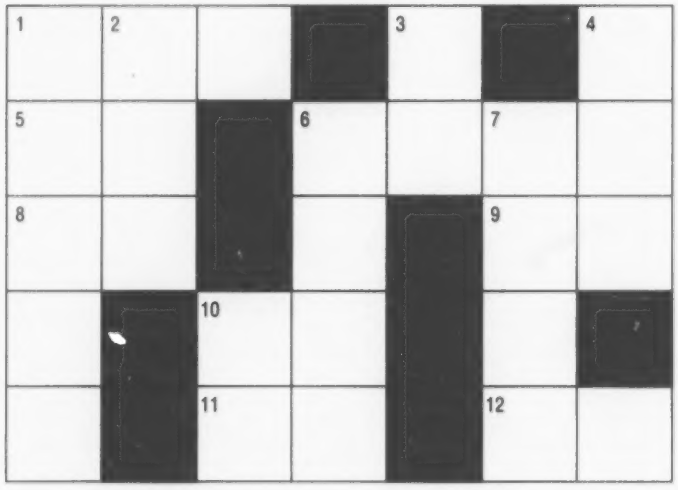
Example: Evaluate for: $a = 2$, $b = 3$, $c = 5$, $d = {}^{-}5$

ACROSS

1. $2(c + d) + abc$
5. $10a + b$
6. $60(b + c) + 70(14 + d)$
8. $c(a + c)$
9. $c^2 + a^2 + c - a - b$
10. $8c + 3(a + b) + d$
11. ${}^{-}10d$
12. $10ac - 1$

DOWN

1. $10\,000 + 1000a + 100b + 10c + d$
2. $100a + 9(b + c) + 9(a + c)$
3. $11 + c + d$
4. $25^2 - 4^2$
6. $(a + b)(a + c)(b + c)(a + b + c + d)$
7. 99 + "6 ACROSS"
10. $(c + d) + ac$



Suggestion: Have students construct a simple cross-number puzzle dealing with these and other skills.

REVIEW ACTIVITIES

Follow the Rule

Intent: A general activity designed to reinforce basic-skill, number-sense, and prealgebra concepts.

Approach: In each sequence the student is given the first number and must then apply the specified rule to obtain the next several terms in the sequence.

Examples:

1. Add 6 to the previous term.
   5, ____, ____, ____, ____, ____

2. Add $^{-}5$ to the previous term.
   15, ____, ____, ____, ____, ____

3. Add 2 1/2 to the previous term.
   4.5, ____, ____, ____, ____, ____

4. Double the previous term, then add 1.
   4, ____, ____, ____, ____, ____

5. Add 1 to the previous term, then double the result.
   4, ____, ____, ____, ____, ____

6. Take the opposite of the previous term.
   $^{-}3$, ____, ____, ____, ____, ____

7. Add $2a$ to the previous term.
   $7a$, ____, ____, ____, ____, ____

8. Double the previous term.
   $a + 3b$, ____, ____, ____, ____, ____

9. Multiply the previous term by $2xy$.
   $3x$, ____, ____, ____, ____, ____

10. Obtain a term by multiplying the preceding two terms.
    6, a, ____, ____, ____, ____, ____

Comments: Students need to be prepared to discuss patterns that result from the rules. A reversal of this activity is the traditional "Guess My Rule" activity.

Example: *ab, 3ab, 9ab, 27ab, 81ab, . . .*

Rule: Multiply previous term by 3.

SUMMARY

This workshop emphasizes aspects of an algebra course that are important for *all* students, independent of their academic level. Among those aspects are—

- engaging students in realistic and interesting problems that require algebraic thinking;
- developing formulas using mathematical processes;
- teacher questioning and student/teacher interaction as a method for students to demonstrate their level of understanding and for the teacher to evaluate that understanding;
- using analogies among algebraic ideas and symbols;
- making lists to solve problems and to aid in thinking algebraically;
- reading algebraic expressions and statements aloud;
- having students communicate about algebra in both oral and written form;
- paying particular attention to the form of student work;
- providing good activities designed to ensure student learning and motivation.

The **NCTM *Curriculum and Evaluation Standards*** contains many statements against which one could measure classroom activities, content, and success. Many of the standards concern the development of algebra. For example, standards on Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections are included at each level, K–4, 5–8, and 9–12. At grades K–4, the standards on Number Sense and Numeration, Patterns and Relationships, and Estimation also are important to the development of algebraic ideas. At the 5–8 level, there is an Algebra standard as well as standards on Number and Number Relationships, Patterns and Functions, and Number Systems and Number Theory. The 9–12 standards include standards for Algebra, Functions, Geometry from an Algebraic Perspective, and Mathematical Structure. These standards should be reviewed while preparing for the workshop to ensure that they are integrated as the workshop proceeds.

Key elements in a successful algebra experience are for students to—

- have good organization in their approach to the subject and to use "good form" when doing their work;
- do their homework consistently and with thought; speak and write so they can see the continuity of the subject;
- be active participants in the course, asking questions and discussing concepts and procedures;
- become engaged in application problems to experience the power of algebra;
- get help early whenever they are in doubt so that needed concepts do not elude them;
- become involved in discussions of tests and quizzes and perform evaluation tasks that are different from traditional tasks (examples are discussing their strengths and weaknesses with their teacher; providing written summaries of what they have learned);
- (above all) develop a positive attitude toward algebra, toward the work they do, and toward themselves.

One of the outcomes of a good algebra course and a solid background in algebraic thinking is mathematical power for the student. Understanding fundamental concepts of algebra provides that power.

There are mathematical themes that should be part of a student's background prior to entering an algebra course. Some of these themes are ordered pairs, plotting points, squares and square roots, evaluation of expressions, working with variables, and solving equations using both informal and formal methods.

Good assignments and high-quality worksheets are key ingredients in helping students develop their algebraic thinking. They may be used to solidify concepts and processes. They should contain worked-out examples preparing students for exercises. They can contribute to the spiral approach, and they provide students a summary of what they have learned. Worksheets in the in-service handbook provide a variety of activities for workshop participants.

BIBLIOGRAPHY AND RESOURCE LIST

Print Materials

1. Edwards, Edgar L., ed. *Algebra for Everyone*. Reston, Va.: National Council of Teachers of Mathematics, 1990. (Note especially the list of resources, pp. 74–89.)
2. Coxford, Arthur F., and Albert P. Shulte, eds. *The Ideas of Algebra, K–12*. 1988 NCTM Yearbook. Reston, Va.: National Council of Teachers of Mathematics, 1988.
3. NCTM Commission on Standards for School Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1989.
4. Lane County Mathematics Project. *Problem Solving in Mathematics, Grade 9*. Palo Alto, Calif.: Dale Seymour Publications, 1983.
5. Foerster, Paul A. *Algebra I: Expressions, Equations, and Applications*. Reading, Mass.: Addison-Wesley Publishing Company, 1990.
6. National Council of Teachers of Mathematics. *The Mathematics Teacher*. A journal devoted to mathematics education, grades 7–14, published monthly, September–May.
7. Educational Testing Service and the College Board. *Algebridge*. Eight units in prealgebra and algebra. Providence, R.I.: Janson Publications, Inc., 1990.
8. Commercial textbooks in prealgebra and algebra.

Manipulative Materials

1. *Algebra Tiles*. Square plastic tiles printed with the numerals 0–9. Available from Creative Publications, 5040 West 111th Street, Oak Lawn, IL 60453.
2. *Algebra Tiles for the Overhead Projector*. Seventy-piece set of color tiles to model rectangular patterns on the overhead projector. Available from Educational Teaching Aids, 199 Carpenter Avenue, Wheeling, IL 60090.

Suppliers of Manipulative Materials

1. Creative Publications, 5040 West 111th Street, Oak Lawn, IL 60453.
2. Educational Teaching Aids, 199 Carpenter Avenue, Wheeling, IL 60090.
3. Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303.

Software

1. *Super Plot*. A function graphing program. Available from Midwest Visual Equipment Company, 6500 North Hamlin, Chicago, IL 60645.
2. *Green Globs and Graphing Equations*. Various games and activities related to plotting points and graphing functions. Available from Sunburst Communications, 101 Castleton Street, Pleasantville, NY 10570-3498.
3. *The Function Supposer: Explorations in Algebra*. Allows students to explore relations among functions, operations on functions, and construction of functions. Available from Sunburst Communications, 101 Castleton Street, Pleasantville, NY 10570-3498.
4. *Algebra Shop*. Students "shop" in stores, using prealgebra and algebra skills. Available from Scholastic, Inc., P.O. Box 7501, 2931 East McCarty Street, Jefferson City, MO 65102.